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HEAT TRANSFER BETWEEN ROTATING ECCENTRIC CYLINDERS WITH DIFFERENT RADII

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Abstract—The regular perturbation solution of heat transfer is developed for eccentric cylinders rotating with different velocities. The solution has been obtained by using a bipolar coordinate system. An attempt has been made to remove the restriction of small clearance ratio from previous work.

NOMENCLATURE

b, characteristic length of the problem;
C_v, specific heat at constant volume;
E, Eckert number, $(R_i^2\Omega_i^2 + R_o^2\Omega_o^2)/[C_v(\bar{T}_1 - \bar{T}_2)]$;
k, thermal conductivity;
L, x_1 coordinate of the centre of the outer cylinder;
Nu, Nusselt number;
Pr, Prandtl number, $\mu C_v/k$;
R_i, radius of inner cylinder;
R_o, radius of outer cylinder;
$\bar{R}$, radius ratio, R_i/R_o;
Re, Reynolds number, $R_o\rho(R_i^2\Omega_i^2 + R_o^2\Omega_o^2)^{1/2}/\mu$;
T, dimensionless temperature;
$\bar{T}$, dimensional temperature;
$\bar{T}_o$, $\bar{T}_i$, temperatures of the outer and inner cylinders, respectively;
u, v, velocity components in ξ and η directions, respectively.

Greek symbols
Ψ, dimensionless stream function;
ξ, η, bipolar co-ordinates;
ε, displacement of the centre of the inner cylinder from the centre of the outer cylinder;
μ, viscosity;
ρ, density;
Ω_o, angular velocity of the outer cylinder;
Ω_i, angular velocity of the inner cylinder;
$\bar{\Omega}V$ velocity ratio, Ω_o/Ω_i.

INTRODUCTION

HEAT transfer in concentric cylinders has been extensively studied due to its wide applications in various engineering devices. A detailed review of the literature has been made [1]. Relatively little work has been reported in the literature for eccentric cylinders. Recently Kuehn and Goldstein [2] conducted an experimental study to determine the influence of eccentricity on natural convective heat transfer through a fluid bounded by two horizontal isothermal cylinders. It was noted that eccentricity alters the local heat transfer on cylinders substantially. Experimental results have also been reported [3]. More recently Yao [4] investigated natural convection in slightly eccentric annuli. The solution procedure can easily be extended to the case when the cylinders are not circular. Singh and Rajvanshi [5] have studied the heat transfer between eccentric cylinders rotating in the same direction and having different temperatures. The energy equation has been transformed into the bipolar coordinate system used by DiPrima and Stuart [6]. Temperature has been obtained as a perturbation solution in terms of the clearance ratio between two cylinders (which is assumed to be small) and the modified Reynolds number. The results are valid for a small clearance ratio and all values of eccentricity.

In the present paper the problem has been investigated further with a view to removing the above restriction on the clearance ratio. The energy equation is expressed in the bipolar coordinate system [7] and shown schematically in Fig. 1. The solution has been obtained by the perturbation method. The results are valid for arbitrary ratios of the radii of the two cylinders. The effects of changes in the eccentricity, and the velocity ratio on temperature profiles and Nusselt number have been discussed in detail.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

Let two eccentric cylinders of radius R_i and $R_o (R_o > R_i)$ with parallel axes contain an incompressible viscous fluid. Both the cylinders are rotating about their respective axis with angular velocities Ω_i and Ω_o respectively. Let the axes of the outer and inner cylinder be at $(L, 0)$ and $(L - \varepsilon, 0)$ respectively, in a rectangular cartesian coordinate system.

Following Ballal and Rivlin [8] we introduce a bipolar coordinate system (ξ, η). It is defined as

$$x_1 = -b\sinh\xi/(\cosh\xi - \cos\eta),\qquad x_2 = b\sin\eta/(\cosh\xi - \cos\eta) \tag{1}$$

where b is a positive number indicating a characteristic length. Let $\xi = \xi_i$ and $\xi = \xi_o$, where ξ_i and ξ_o are negative constants, be the surfaces of the inner and

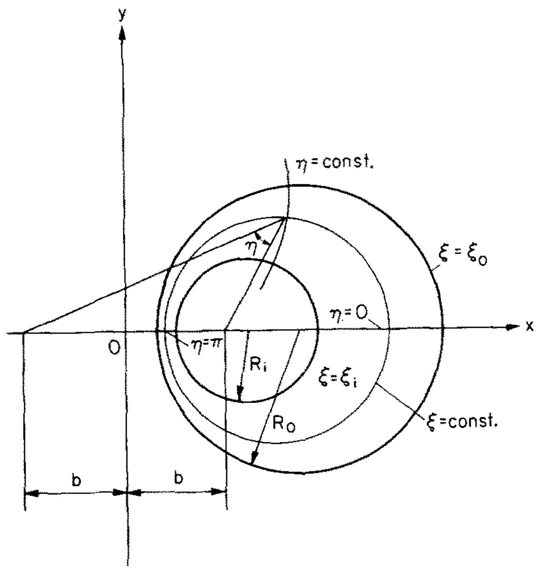

FIG. 1. Bipolar co-ordinate system.

outer cylinders, respectively. R_i, R_o, L and ε are defined as

$$R_i = -b/\sinh\xi_i,\ R_o = -b/\sinh\xi_o,\qquad \varepsilon = -b(\coth\xi_o - \coth\xi_i),\ L = -b\coth\xi_o. \tag{2}$$

From equation (2) we obtain

$$b = [(R_i^2 + R_o^2 - \varepsilon^2)^2 - 4R_i^2R_o^2]^{1/2}/(2\varepsilon). \tag{3}$$

Let u and v be the velocity components in the ξ and η directions respectively, $\bar{T}$ be the temperature at any point (ξ, η), $\bar{T}_i$ and $\bar{T}_o$ be the constant temperatures of the inner and outer cylinders respectively. The boundary conditions are

$$u = 0,\ v = R_i\Omega_i,\ \bar{T} = \bar{T}_i \ \text{ at } \xi = \xi_i, \tag{4}$$

$$u = 0,\ v = R_o\Omega_o,\ \bar{T} = \bar{T}_o \ \text{ at } \xi = \xi_o. \tag{5}$$

We define the dimensionless stream function ψ and temperature T in the following form

$$(R_i^2\Omega_i^2 + R_o^2\Omega_o^2)^{1/2}(\partial\Psi/\partial\eta) = (H/R_o)u,\qquad (R_i^2\Omega_i^2 + R_o^2\Omega_o^2)^{1/2}(\partial\Psi/\partial\xi) = -(H/R_o)v, \tag{6}$$

$$T = (\bar{T} - \bar{T}_i)/(\bar{T}_o - \bar{T}_i)$$

where

$$H = b/(\cosh\xi - \cos\eta) \tag{7}$$

and

$(\bar{T}_o - \bar{T}_i)$ will be positive or negative, as $\bar{T}_o \gtrless \bar{T}_i$.

Using equation (6) the steady state energy equation for the 2-dim., viscous, incompressible flow in the (ξ, η) coordinate system may be written as

$$(\partial^2T/\partial\xi^2) + (\partial^2T/\partial\eta^2) = Pr\,Re[(\partial\xi/\partial\eta)(\partial T/\partial\xi) - (\partial\Psi/\partial\xi)(\partial T/\partial\eta)] - Pr\,E[4Y_1^2 + Y_2^2], \tag{8}$$

such that

$$Y_1 = [(b/H)(\partial^2\Psi/\partial\xi\partial\eta) + \sinh\xi(\partial\Psi/\partial\eta) + \sin\eta(\partial\Psi/\partial\xi)](R_o/b),$$

$$Y_2 = (R_o/b)\{(b/H)[(\partial^2\Psi/\partial\eta^2) - (\partial^2\Psi/\partial\xi^2)] + 2\sin\eta(\partial\Psi/\partial\eta) - 2\sinh\xi(\partial\Psi/\partial\xi)\}.$$

Boundary conditions (4) and (5) for temperature reduce to

$$T = 0 \quad \text{at } \xi = \xi_i, \tag{9}$$

$$T = 1 \quad \text{at } \xi = \xi_o. \tag{10}$$

For slow motion solution, Ψ and T are assumed in the form

$$[\Psi, T] = [\Psi^{(0)}T^{(0)}] + Re[\Psi^{(1)}T^{(1)}] + O(Re^2). \tag{11}$$

Equation (11) is substituted in equation (8) and the coefficients of like powers of Re are equated. The first two equations are as follows:

$$(\partial^2T^{(0)}/\partial\xi^2) + (\partial^2T^{(0)}/\partial\eta^2) = -Pr\,E[4Y_1^{(0)2} + Y_2^{(0)2}], \tag{12}$$

$$(\partial^2T^{(1)}/\partial\xi^2) + (\partial^2T^{(1)}/\partial\eta^2) = Pr[(\partial\Psi^{(0)}/\partial\eta)(\partial T^{(0)}/\partial\xi) - (\partial\Psi^{(0)}/\partial\xi)(\partial T^{(0)}/\partial\eta)] - Pr\,E[8Y_1^{(0)}Y_1^{(1)} + 2Y_2^{(0)}Y_2^{(1)}] \tag{13}$$

where

$$Y_1^{(0)} = (R_o/b)[(b/H)(\partial^2\Psi^{(0)}/\partial\xi\partial\eta) + \sinh\xi(\partial\Psi^{(0)}/\partial\eta) + \sin\eta(\partial\Psi^{(0)}/\partial\xi)],$$

$$Y_1^{(1)} = (R_o/b)[(b/H)(\partial^2\Psi^{(1)}/\partial\xi\partial\eta) + \sinh\xi(\partial\Psi^{(1)}/\partial\eta) + \sin\eta(\partial\Psi^{(1)}/\partial\xi)],$$

$$Y_2^{(0)} = (R_o/b)\{(b/H)[(\partial^2\Psi^{(0)}/\partial\eta^2) - (\partial^2\Psi^{(0)}/\partial\xi^2)] + 2\sin\eta(\partial\Psi^{(0)}/\partial\eta) - 2\sinh\xi(\partial\Psi^{(0)}/\partial\xi)\}$$

and

$$Y_2^{(1)} = (R_o/b)\{(b/H)[(\partial^2\Psi^{(1)}/\partial\eta^2) - (\partial^2\Psi^{(1)}/\partial\xi^2)] + 2\sin\eta(\partial\Psi^{(1)}/\partial\eta) - 2\sinh\xi(\partial\Psi^{(1)}/\partial\xi)\}.$$

The boundary conditions (9) and (10) take the form

$$\left.\begin{aligned} T^{(0)} &= 0 \quad \text{at } \xi = \xi_i, \\ T^{(0)} &= 1 \quad \text{at } \xi = \xi_o, \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} T^{(1)} &= 0 \quad \text{at } \xi = \xi_i, \\ T^{(1)} &= 0 \quad \text{at } \xi = \xi_o, \end{aligned}\right\} \tag{15}$$

SOLUTION OF EQUATIONS

$\Psi^{(0)}$ is taken from [8] as

$$\Psi^{(0)} = H[F_0(\xi) + F_1(\xi)\cos\eta], \tag{16}$$

where

$$F_0(\xi) = (A_0 + C_0\xi)\cosh\xi + (B_0 + D_0\xi)\sinh\xi, \tag{17}$$

$$F_1(\xi) = A_1\cosh 2\xi + B_1\sinh 2\xi + C_1\xi + D_1, \tag{18}$$

$$(A_0, B_0, C_0, D_0) = (f_1, f_3, f_5, f_7)\alpha + (f_2, f_4, f_6, f_8)\beta, \tag{19}$$

$$(A_1, B_1, C_1, D_1) = (f_9, f_{11}, -f_5, f_{13})\alpha + (f_{10}, f_{12}, -f_6, f_{14})\beta,$$

such that

$$\left.\begin{aligned}\alpha &= \bar{\Omega}/[R_o(\bar{\Omega}^2 + \bar{R}^2)^{1/2}],\\ \beta &= \bar{R}/R_o(\bar{\Omega}^2 + \bar{R}^2)^{1/2}]\end{aligned}\right\} \tag{20}$$

$$\bar{\Omega}\ \text{(angular velocity ratio)} = \Omega_o/\Omega_i, \tag{21}$$

$$\bar{R}\ \text{(radii ratio)} = R_i/R_o. \tag{22}$$

The quantities f_j ($j = 1 - 14$) are defined in ref. [8] and have not been recorded here for the sake of brevity.

We assume $T^{(0)}$ in the following form:

$$T^{(0)}(\xi, \eta) = Pr\,ER_o^2 \sum_{n=0}^{2} S_n(\xi)\cos n\eta. \tag{23}$$

Using equations (16) and (23) in equation (12) and equating the terms independent of η, the coefficients of $\cos\eta$ and $\cos 2\eta$ in both sides we obtain the following set of equations:

$$S_0'' = -2F_1'^2 - (F_0 - F_0'')^2 - \tfrac{1}{2}F_1'^2, \tag{24}$$

$$S_1'' - S_1 = -2F_1''(F_0'' - F_0), \tag{25}$$

$$S_2'' - 4S_2 = 2F_1'^2 - \tfrac{1}{2}F_1''^2, \tag{26}$$

where a prime denotes differentiation with respect to ξ.

Boundary conditions (14) take the form

$$S_0 = 0,\ \text{at}\ \xi = \xi_i,\ S_0 = 1/(Pr\,ER_o^2),\ \text{at}\ \xi = \xi_o, \tag{27}$$

$$S_1 = 0,\ \text{at}\ \xi = \xi_i,\ S_1 = 0\ \text{at}\ \xi = \xi_o, \tag{28}$$

$$S_2 = 0,\ \text{at}\ \xi = \xi_i,\ S_2 = 0\ \text{at}\ \xi = \xi_o. \tag{29}$$

The solution to equation (24), subjected to boundary conditions (27), is given by

$$S_0(\xi) = [\varphi_0(\xi_i)(\xi_o - \xi) + \varphi_0(\xi_o)(\xi - \xi_i) + (1/Pr\,ER_o^2)(\xi - \xi_i)]/(\xi_o - \xi_i) - \varphi_0(\xi) \tag{30}$$

where

$$\begin{aligned}\varphi_0(\xi) &= 0.5(A_1^2 + B_1^2)\cosh 4\xi + A_1B_1\sinh 4\xi\\ &\quad + 0.5(4B_1C_1 + C_0^2 + D_0^2)\cosh 2\xi\\ &\quad + (2A_1C_1 + C_0D_0)\sinh 2\xi\\ &\quad + (C_1^2 - C_0^2 + D_0^2)\xi^2.\end{aligned}$$

The solution of (25) under boundary conditions (28) is represented by

$$S_1(\xi) = [\varphi_1(\xi_o)\sinh(\xi - \xi_i) + \varphi_1(\xi_i)\sinh(\xi_o - \xi)] \times \operatorname{cosech}(\xi_o - \xi_i) - \varphi_1(\xi) \tag{31}$$

where

$$\begin{aligned}\varphi_1(\xi) &= (A_1D_0 + B_1C_0)\cosh 3\xi\\ &\quad + (A_1C_0 + B_1D_0)\sinh 3\xi\\ &\quad + 4\xi[(B_1D_0 - A_1C_0)\cosh\xi\\ &\quad + (A_1D_0 - B_1C_0)\sinh\xi].\end{aligned}$$

The solution of (26) under boundary conditions (29) is obtained as

$$S_2(\xi) = [\varphi_2(\xi_o)\sinh 2(\xi_i - \xi) + \varphi_2(\xi_i) \times \sinh 2(\xi - \xi_o)]\operatorname{cosech} 2(\xi_o - \xi_i) + \varphi_2(\xi) \tag{32}$$

where

$$\varphi_2(\xi) = \xi[2A_1C_1\cosh 2\xi + 2B_1C_1\sinh 2\xi] + 2(A_1^2 - B_1^2) - \tfrac{1}{2}C_1^2.$$

$\Psi^{(1)}$ is taken from ref. [8] as

$$\Psi^{(1)} = 2bH\sum_{m=1}^{\infty} G_m(\xi)\sin m\eta. \tag{33}$$

We assume $T^{(1)}$ as under

$$T^{(1)}(\xi, \eta) = Pr\,ER_o^2 b\sum_{m=1}^{\infty} J_m(\xi)\sin m\eta. \tag{34}$$

Equations (13), (33) and (34), the orthogonality relations for trignometric functions and the relations

$$\int_0^{2\pi}[\cos r\eta/(\cosh\xi - \cos\eta)^2]\mathrm{d}\eta = 2\pi\exp(r\xi)(r - \coth\xi)/\sinh^2\xi$$

give

$$J_m'' - m^2J_m = Pr\,K_m(\xi) - I_m(\xi) \tag{35}$$

where

$$K_m(\xi) = \sum_{n=1}^{4}\{\exp[(m+n)\xi](m - n - \coth\xi) - \exp[(m-n)\xi](m + n - \coth\xi)\}/(R_n\sinh^2\xi),$$

$$\begin{aligned}I_m(\xi) &= 4(F_0'' - F_0)[G_m'' + (m^2 - 1)G_m]\\ &\quad + 8(m+1)F_1'G_{m+1}' - 8(m-1)F_1'G_{m-1}'\\ &\quad + 2F_1''[G_{m+1}'' + m(m+2)G_{m+1}]\\ &\quad + 2F_1''[G_{m-1}'' + m(m-2)G_{m-1}],\end{aligned}$$

$$R_1(\xi) = (E_0 + 0.25F_1')S_1 + E_1S_2 + E_2(0.5S_2' - S_0'),$$

$$R_2(\xi) = 2E_0S_2 + 0.5(E_1S_1 - E_2S_1'),$$

$$R_3(\xi) = E_1S_2 - 0.25(S_1F_1' + 2E_2S_2'),$$

$$R_4(\xi) = -0.5F_1'S_2,$$

$$E_0(\xi) = F_0'\cosh\xi - F_0\sinh\xi - 0.5F_1',$$

$$E_1(\xi) = F_1'\cosh\xi - F_1\sinh\xi - F_0',$$

$$E_2(\xi) = F_0 + F_1\cosh\xi.$$

The functions $G_m(\xi)$ are defined in ref. [8] and are not recorded here for sake of brevity. The boundary conditions (15) change to

$$J_m = 0, \quad \text{at } \xi = \xi_i, \qquad J_m = 0, \quad \text{at } \xi = \xi_o. \tag{36}$$

The solution of equation (35) subjected to boundary conditions (36) is given by

$$\begin{aligned} J_m(\xi) = Pr\{&[\varphi_3(\xi_o)\sinh m(\xi_i - \xi) \\ &+ \varphi_3(\xi_i)\sinh m(\xi - \xi_o)]\ \mathrm{cosech}\ m(\xi_o - \xi_i) \\ &+ \varphi_3(\xi)\} + \{[\varphi_4(\xi_o)\sinh m(\xi - \xi_i) \\ &+ \varphi_4(\xi_i)\sinh m(\xi_o - \xi)]\ \mathrm{cosech}\ m(\xi_o - \xi_i) \\ &- \varphi_4(\xi)\}. \end{aligned} \tag{37}$$

where

$$\varphi_3(\xi) = \exp(m\xi)\int \exp(-2m\xi)\ [\int \exp(m\xi)K_m(\xi)\mathrm{d}\xi]\mathrm{d}\xi$$

$$\varphi_4(\xi) = \exp(m\xi)\int \exp(-2m\xi)\ [\int \exp(m\xi)I_m(\xi)\mathrm{d}\xi]\mathrm{d}\xi.$$

It may be seen from the above analysis that no restriction has been placed on the eccentricity or the clearance ratio in obtaining the solution, which is valid for slow motion only. Various problems do arise when one attempts to obtain a solution, at $\bar{\varepsilon} = 0$ and 1. When $\bar{\varepsilon} \to 0$, b and $\sinh \xi$ both approach infinity, so that the ratio is finite. The problem reduces to that of concentric cylinders. The second limit, that is, $\bar{\varepsilon} \to 1$ means $b \to 0$. This reduces the problem to that of two eccentric cylinders in contact at one point. However, no attempt has been made in this study to derive the results for these cases.

TEMPERATURE PROFILES

The important parameters of the problem are $\bar{\varepsilon}$, $\bar{\Omega}$, $\bar{R}$, Pr and E, where $\bar{\varepsilon}$ is given by

$$\bar{\varepsilon} = \varepsilon/(R_o - R_i). \tag{38}$$

We further define

$$\bar{\xi} = (\xi - \xi_i)/(\xi_o - \xi_i). \tag{39}$$

For numerical work we take

$$Pr = 5.0,\ R_o = 1.0,\ E = 0.1.$$

R_i and ε are evaluated for a given set of values of $\bar{R}$ and $\bar{\varepsilon}$. Using these results b is calculated from equation (3). ξ_i and ξ_o are evaluated using equation (2). Then the functions f_i (i = 1–14) are evaluated using definitions recorded in ref. [8]. At this stage $\bar{\Omega}$ is also assigned a value and the constants A_0, B_0, C_0, D_0, A_1, B_1, C_1 and D_1 are calculated using equations (19) and (20). Using these values the other constants occurring in the solution are evaluated. The temperature profiles are now obtained from equations (23) and (34) by using assigned values of Pr and E. Various values were assigned to the parameters $\bar{\varepsilon}$, $\bar{\Omega}$ and $\bar{R}$ and different sets of results were obtained. Table 1 depicts the temperature profiles for various values of $\bar{R}$ and for fixed values of other parameters, at $\eta = 0$ (the point of maximum clearance). Figure 2 shows the temperature profiles for various values of $\bar{\varepsilon}$ when outer cylinder is stationary. Figure 3 shows the same when both the cylinders are rotating. With increase in $\bar{\varepsilon}$ the convective effects dominate over the temperature distribution. This makes the temperature profile more curved. The effect becomes negligible in the region of minimum clearance ($\eta = \pi$), where the gap is small.

NUSSELT NUMBER

The heat exchange between the cylinders and the fluid is measured by the local heat transfer coefficient.

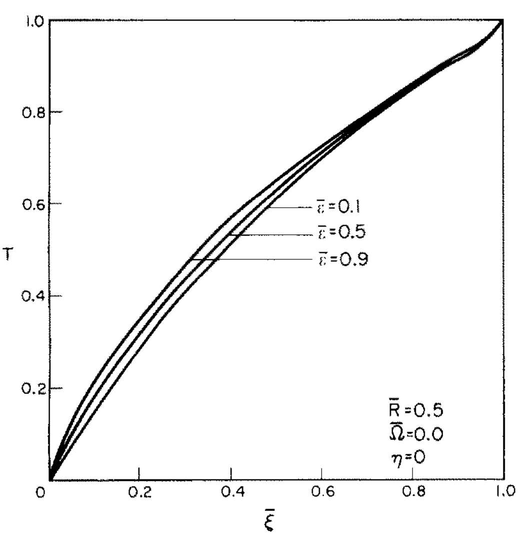

FIG. 2. Temperature profiles for various $\bar{\varepsilon}$ when outer cylinder is stationary.

Table 1. Temperature profiles for various $\bar{R}$ at Pr = 5.0, $\bar{\Omega}$ = 0.0, $\bar{\varepsilon}$ = 0.2, η = 0

$\bar{R}$	$\bar{\xi}$ = 0.0	0.2	0.4	0.6	0.8	1.0
0.1	0.0000	0.4117	0.6308	0.7756	0.8924	1.0000
0.2	0.0000	0.3639	0.5912	0.7520	0.8820	1.0000
0.3	0.0000	0.3341	0.5621	0.7320	0.8723	1.0000
0.4	0.0000	0.3130	0.5395	0.7153	0.8637	1.0000
0.5	0.0000	0.2972	0.5216	0.7015	0.8565	1.0000
0.6	0.0000	0.2848	0.5072	0.6900	0.8503	1.0000
0.7	0.0000	0.2748	0.4952	0.6804	0.8451	1.0000
0.8	0.0000	0.2667	0.4852	0.6722	0.8406	1.0000
0.9	0.0000	0.2599	0.4768	0.6652	0.8368	1.0000

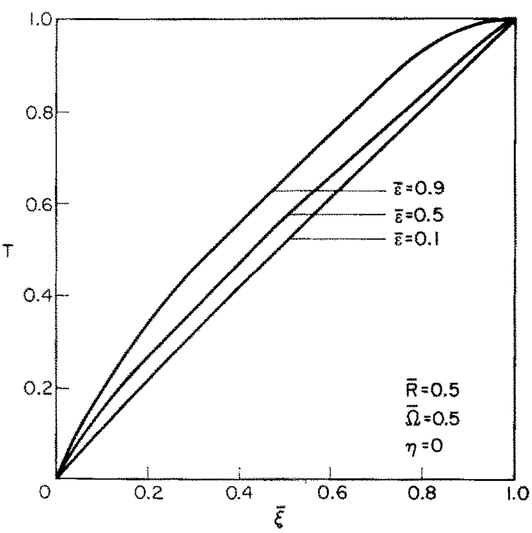

FIG. 3. Temperature profiles for various $\bar{\varepsilon}$ when both the cylinders are rotating.

Once the temperature distribution around the cylinders is known, the local heat transfer coefficient can be evaluated. The Nusselt number at any boundary is defined as

$$Nu = -[L_1/(\bar{T}_w - \bar{T}_f)][\partial \bar{T}/\partial N]_{N=0}, \quad (40)$$

where $\bar{T}_w$ is the wall temperature, $\bar{T}_f$ is the free steam temperature, L_1 is a characteristic length and dN is the length element in the normal direction. Taking $\bar{T}_w = \bar{T}_i$, $\bar{T}_f = \bar{T}_o$ the temperature of the outer cylinder, $L_1 = b$ and $dN = H\,d\xi$ and using equation (6) the Nusselt number at the inner cylinder is given by

$$(Nu)_i = [(b/H)(\partial T/\partial \xi)]_{\xi=\xi_i}. \quad (41)$$

Using equations (11), (23) and (34) in equation (40) we get

$$(Nu)_i = Pr\,R_o^2 E\left\{(b/H)_{\xi=\xi_i}\left[\sum_{n=0}^{2} S_n'(\xi_i)\cos n\eta + Re\,b\sum_{m=1}^{\infty} J_m'(\xi_i)\sin m\eta\right]\right\}. \quad (42)$$

The average Nusselt number on the inner cylinder is defined as

$$(Nu^*)_i = (1/2\pi R_i)\int_0^{2\pi}(Nu)_i(H)_{\xi=\xi_i}\,d\eta. \quad (43)$$

Using equation (41), equation (42) takes the form

$$(Nu^*)_i = Pr\,ER_o^2 bS_o'(\xi_i)/R_i. \quad (44)$$

Similarly the average Nusselt number on the outer cylinder is obtained as

$$(Nu^*)_o = Pr\,ER_o bS_o'(\xi_o). \quad (45)$$

Tables 2 and 3 show the average Nusselt number on the inner and outer cylinder respectively for varying conditions of $\bar{\varepsilon}$ and $\bar{R}$. Figure 4 shows the variations of the average Nusselt number against $\bar{\varepsilon}$ for various values of the velocity ratio $\bar{\Omega}$. Figure 5 depicts the average Nusselt number against $\bar{R}$ for various values of $\bar{\Omega}$.

Table 2. Average Nusselt number on the inner cylinder for varying values of $\bar{R}$ and $\bar{\varepsilon}$, $Pr = 5.0$, $\bar{\Omega} = 0.0$

	$\bar{\varepsilon}$								
$\bar{R}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	67.4486	32.9880	21.1729	15.0181	11.1264	8.3654	6.2499	4.5378	3.0964
0.2	41.2864	20.3462	13.2323	9.5731	7.2940	5.7034	4.5050	3.5511	2.7598
0.3	34.1840	16.9580	11.1533	8.2008	6.3860	5.1374	4.2102	3.4822	2.8858
0.4	32.5085	16.2166	10.7647	8.0180	6.3489	5.2147	4.3829	3.7378	3.2154
0.5	33.9075	16.9917	11.3636	8.5509	6.8582	5.7202	4.8948	4.2616	3.7542
0.6	38.3403	19.2840	12.9736	9.8409	7.9710	6.7252	5.8303	5.1503	4.6105
0.7	47.4973	23.9601	16.1958	12.3625	10.0898	8.5873	7.5167	6.7097	6.0742
0.8	67.4590	34.1086	23.1417	17.7514	14.5733	12.4855	11.0079	9.9015	9.0358
0.9	129.5147	65.6056	41.6430	34.3743	28.3483	24.4123	21.6389	19.5749	17.9647

Table 3. Average Nusselt number on the outer cylinder for varying values of $\bar{R}$ and $\bar{\varepsilon}$, $Pr = 5.0$, $\bar{\Omega} = 0.0$

	$\bar{\varepsilon}$								
$\bar{R}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.2335	0.6095	0.3982	0.2900	0.2330	0.1765	0.1417	0.1143	0.0920
0.2	2.0437	1.0171	0.6727	0.4986	0.3925	0.3203	0.2673	0.2263	0.1935
0.3	3.1380	1.5675	1.0429	0.7793	0.6199	0.5121	0.4337	0.3735	0.3254
0.4	4.6804	2.3413	1.5613	1.1704	0.9346	0.7758	0.6606	0.5724	0.5022
0.5	6.9417	3.4729	2.3165	1.7373	1.3881	1.1533	0.9833	0.8537	0.7508
0.6	10.4573	5.2285	3.4844	2.6103	2.0833	1.7291	1.4733	1.2788	1.1251
0.7	16.4748	8.2290	5.4754	4.0940	3.2607	2.7009	2.2973	1.9912	1.7504
0.8	28.7386	14.3367	9.5215	7.1027	5.6424	4.6618	3.9557	3.4218	3.0035
0.9	65.9632	32.8641	21.7833	16.2076	12.8389	10.5789	8.9518	7.7253	6.7666

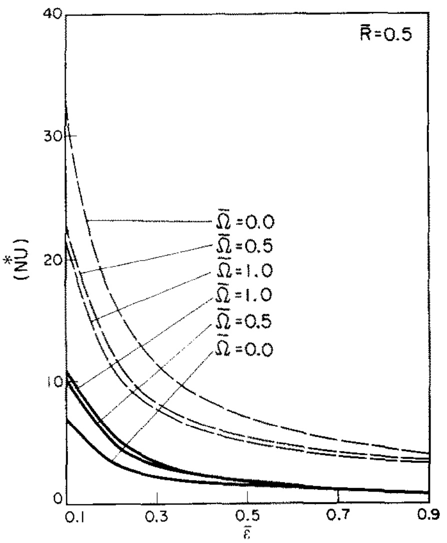

FIG. 4. Average Nusselt number against $\bar{\varepsilon}$ for various $\bar{\Omega}$, outer cylinder —— inner cylinder - - -.

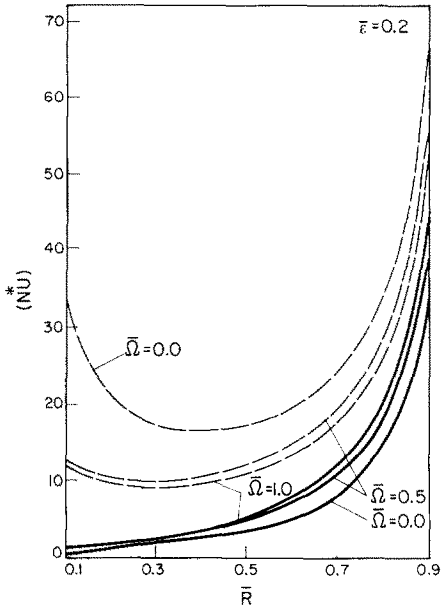

FIG. 5. Average Nusselt number against $\bar{R}$ for various $\bar{\Omega}$, outer cylinder —— inner cylinder - - -.

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TRANSFERT THERMIQUE ENTRE DES CYLINDRES EXCENTRES ET TOURNANTS, AVEC DES RAYONS DIFFERENTS

Résumé—On présente la solution de perturbation régulière du transfert de chaleur entre des cylindres excentrés et tournant à des vitesses différentes. La solution est obtenue en utilisant un système de coordonnées bipolaire. Un essai est fait pour éliminer la restriction d'un petit rapport d'espacement faite précédement par les auteurs.

WÄRMEÜBERTRAGUNG ZWISCHEN EXZENTRISCHEN ROTIERENDEN ZYLINDERN MIT UNTERSCHIEDLICHEN RADIEN

Zusammenfassung—Die reguläre Störungslösung der Wärmeübertragung wird für exzentrische, mit unterschiedlichen Geschwindigkeiten rotierende Zylinder entwickelt. Die Lösung wurde unter Verwendung eines bipolaren Koordinatensystems erhalten. Dabei wurde gegenüber einer früheren Arbeit des Autors angestrebt, die Einschränkung der Gültigkeit auf kleine Spaltweiten fallen zu lassen.

ТЕПЛОПЕРЕНОС МЕЖДУ ВРАЩАЮЩИМИСЯ ЭКСЦЕНТРИЧЕСКИМИ ЦИЛИНДРАМИ С РАЗЛИЧНЫМИ РАДИУСАМИ

Аннотация—Методом возмущений получено решение задачи о теплообмене эксцентрических цилиндров, вращающихся с различными скоростями. Решение получено с использованием биполярной системы координат. Предпринята попытка распространить теорию на большие значения зазора в отличие от предыдущей работы авторов.